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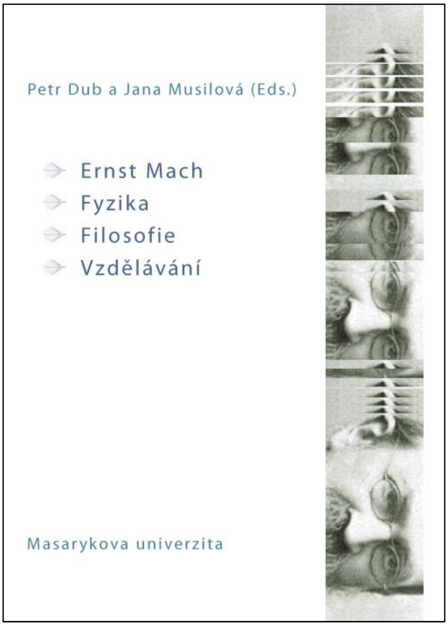
INFLUENCE OF THE DARK ENERGY ON ASTROPHYSICAL PHENOMENA IN ACTIVE GALACTIC NUCLEI

The cosmological constant $\Lambda_0>0$ reflecting vacuum energy represents one of the strongest links between Einstein's General Relativity and Mach's ideas that substantially inspired the Einstein theory. A wide variety of cosmological tests indicate accelerated expansion of the recent universe caused by a dark energy close to $\Lambda_0 \approx 1.3\times10^{-56}$ m^{-2}. Surprisingly, such a small repulsive cosmological constant has a crucial influence on accretion discs around supermassive black holes in giant galaxies. There is an outer edge of the discs allowing outflow of matter into outer space. Extension of discs in quasars is comparable with extension of the associated galaxies suggesting that observed relict Λ_0 puts an upper limit on the galaxy extension. Jets produced in the innermost parts of accretion discs can be significantly collimated due to the cosmic repulsion after leaving the galaxy.



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Influence of the Dark Energy on Astrophysical Phenomena in Active Galactic Nuclei

Zdeněk Stuchlík

Introduction

It is well known that while inventing the General Relativity, Albert Einstein was strongly inspired by ideas of Ernst Mach related to the origin of the inertial forces. The extent of agreement between General Relativity and Mach's ideas is a matter of long discussion. At the present time, there are two widely debated relativistic effects of the Machian origin: dragging of inertial frames and cosmological constant. The present work is devoted to the influence of the repulsive cosmological constant in astrophysical processes related to accretion discs orbiting supermassive black holes in nuclei of giant galaxies.

The repulsive cosmological constant was introduced by A. Einstein into the gravitational equations in order to construct a model of static universe. When expansion of the Universe was confirmed by Hubble's observations, Einstein pronounced introducing Λ to be a big mistake. However, history is ironic – at present time the cosmological constant and its modifications called dark energy represent a crucial ingredient of the cosmological models both in the recent state and in the introductory, inflationary phase. The Machian origin of the cosmological constant was fully enlightened by Yakov Zel'dovich, who showed that Λ can be identified with the energy density of vacuum.

Data from cosmological tests indicate convincingly that within the framework of the inflationary cosmology a non-zero, although very small, vacuum energy density, i.e., a relict repulsive cosmological constant (RRCC), $\Lambda > 0$, or some kind of similarly acting dark energy, has to be invoked in order to explain the dynamics of the recent Universe [1, 2]. There is a strong „concordance" indication [3] that the observed value of the vacuum energy density is $\varrho_{\mathrm{vac}(0)} \approx 0.73\,\varrho_{\mathrm{crit}(0)}$ with present values of the critical energy density $\varrho_{\mathrm{crit}(0)}$, and the Hubble parameter H_0 given by $\varrho_{\mathrm{crit}(0)} = 3H_0^2/8\pi$, $H_0 = 100h\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. Taking value of the dimensionless parameter $h \approx 0.7$, we obtain the RRCC to be $\Lambda_0 = 8\pi\varrho_{\mathrm{vac}(0)} \approx 1.3 \times 10^{-56}\,\mathrm{m^{-2}}$. It is well known that the RRCC strongly influences expansion of the Universe, leading finally to an exponentially accelerated stage [4]. However, we show that the RRCC can be relevant for accretion processes in the field of central black holes in quasars and active galactic nuclei. Basic properties

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of geometrically thin accretion discs with low accretion rates and negligible pressure are given by the circular geodetical motion in the black-hole backgrounds [5], while for geometrically thick discs with high accretion rates and relevant pressure they are determined by equipotential surfaces of test perfect fluid rotating in the backgrounds [6, 7]. The presence of the RRCC changes substantially the asymptotic structure of the black-hole (naked-singularity) backgrounds as they become asymptotically de Sitter and contain a cosmological event horizon behind which the spacetime is dynamic. We shall discuss the role of the RRCC in astrophysically most relevant, rotating backgrounds.

Kerr–de Sitter spacetimes

In the standard Boyer–Lindquist coordinates $(t, r, \vartheta, \varphi)$ and the geometric units ($c = G = 1$), the Kerr–de Sitter (KdS) geometry is given by the line element

$$\begin{aligned} \mathrm{d}s^2 = &- \frac{\Delta_r}{I^2 \varrho^2}(\mathrm{d}t - a\sin^2\vartheta\,\mathrm{d}\varphi)^2 + \frac{\Delta_\vartheta \sin^2\vartheta}{I^2\varrho^2}\left[a\,\mathrm{d}t - (r^2+a^2)\,\mathrm{d}\varphi\right]^2 \\ &+ \frac{\varrho^2}{\Delta_r}\,\mathrm{d}r^2 + \frac{\varrho^2}{\Delta_\vartheta}\,\mathrm{d}\vartheta^2\,, \end{aligned} \tag{1}$$

$$\Delta_r = -\tfrac{1}{3}\Lambda r^2(r^2+a^2) + r^2 - 2Mr + a^2\,,$$
$$\Delta_\vartheta = 1 + \tfrac{1}{3}\Lambda a^2\cos^2\vartheta\,, \quad I = 1 + \tfrac{1}{3}\Lambda a^2\,, \quad \varrho^2 = r^2 + a^2\cos^2\vartheta\,.$$

The parameters of the spacetime are: mass (M), specific angular momentum (a), and cosmological constant (Λ). It is convenient to introduce a dimensionless cosmological parameter $y = \frac{1}{3}\Lambda M^2$. For simplicity, we put $M = 1$ hereafter. The event horizons of the spacetime are given by the pseudosingularities of the line element (1), determined by the condition $\Delta_r = 0$. The loci of the event horizons are implicitly determined by the relation

$$a^2 = a^2_{\mathrm{h}}(r; y) \equiv \frac{r^2 - 2r - yr^4}{yr^2 - 1}\,.$$

It can be shown [8] that a critical value of the cosmological parameter exists $y_{\mathrm{c(KdS)}} = 16/(3+2\sqrt{3}\,)^3 \doteq 0.05924$, such that for $y > y_{\mathrm{c(KdS)}}$, only naked-singularity backgrounds exist for $a^2 > 0$. There is another critical value $y_{\mathrm{c(SdS)}} = 1/27 \doteq 0.03704$, which is limiting the existence of Schwarzschild–de Sitter (SdS) black holes [9]. In the Reissner–Nordström–de Sitter (RNdS) spacetimes, the critical value is $y_{\mathrm{c(RNdS)}} = 2/27 \doteq 0.07407$ [10].

Thin discs

Basic properties of thin accretion discs are determined by equatorial circular motion of test particles because any tilted disc has to be driven to the equatorial plane of the rotating spacetimes due to the dragging of inertial frames [11].

The motion of a test particle with rest mass m is given by the geodesic equations. In a separated and integrated form, the equations were obtained by Carter [12]. For the motion restricted to the equatorial plane ($\mathrm{d}\vartheta/\mathrm{d}\lambda = 0$, $\vartheta = \pi/2$) of the KdS spacetime, the Carter equations take the following form

$$r^2\frac{\mathrm{d}r}{\mathrm{d}\lambda} = \pm R^{1/2}(r)\,, \quad r^2\frac{\mathrm{d}\varphi}{\mathrm{d}\lambda} = -IP_\vartheta + \frac{aIP_r}{\Delta_r}\,, \quad r^2\frac{\mathrm{d}t}{\mathrm{d}\lambda} = -aIP_\vartheta + \frac{(r^2+a^2)IP_r}{\Delta_r}\,,$$

where

$$R(r) = P_r^2 - \Delta_r(m^2r^2 + K)\,, \quad P_r = I\mathcal{E}(r^2+a^2) - Ia\Phi\,, \quad P_\vartheta = I(a\mathcal{E} - \Phi)\,,$$
$$K = I^2(a\mathcal{E} - \Phi)^2\,. \tag{2}$$

The proper time of the particle τ is related to the affine parameter λ by $\tau = m\lambda$. The constants of motion are: energy ($\mathcal{E}$), axial angular momentum (Φ), „total“ angular momentum (K). For the equatorial motion, K is restricted through Eq. (2) following from the conditions on the latitudinal motion [13]. Notice that $\mathcal{E}$ and Φ cannot be interpreted as energy and axial angular momentum at infinity, since the spacetime is not asymptotically flat. The condition $R(r) = 0$ determines the effective potential $E_+(r; L, a, y)$ and motion is allowed where $E \geq E_+$ [14].

The equatorial circular orbits can be determined by solving simultaneously the equations $R(r) = 0$, $\mathrm{d}R/\mathrm{d}r = 0$. The specific energy and angular momentum of the orbits are

$$E_\pm(r; a, y) = \frac{1 - \frac{2}{r} - (r^2+a^2)y \pm a\left(\frac{1}{r^3} - y\right)^{1/2}}{\left[1 - \frac{3}{r} - a^2y \pm 2a\left(\frac{1}{r^3} - y\right)^{1/2}\right]^{1/2}}\,, \tag{3}$$

$$L_\pm(r; a, y) = -\frac{2a + ar(r^2+a^2)y \mp r(r^2+a^2)\left(\frac{1}{r^3} - y\right)^{1/2}}{r\left[1 - \frac{3}{r} - a^2y \pm 2a\left(\frac{1}{r^3} - y\right)^{1/2}\right]^{1/2}}\,. \tag{4}$$

The relations (3)–(4) determine two families of the circular orbits. We call them plus-family orbits and minus-family orbits [8] according to the $\pm$ sign in the relations (3)–(4). Inspecting expressions (3) and (4), we find two reality restrictions on the circular orbits. The first one introduces the notion of the „static radius“, given by the formula $r_\mathrm{s} = y^{-1/3}$ independently of the rotational parameter a. It can be compared with formally identical result in the Schwarzschild–de Sitter spacetimes [9]. A „free“ or „geodetical“ observer on the static radius has only U^t component of 4-velocity being non-zero, the gravitational attraction of the black hole is just balanced by the cosmic repulsion there. The position on the static radius is unstable relative to radial perturbations. The second restriction is given by the condition $1 - 3/r - a^2y \pm 2a(r^{-3} - y)^{1/2} \geq 0$ the equality determines photon circular orbits with $E \to \infty$ and $L \to \pm\infty$. A detailed discussion of the photon circular orbits can be found in Refs [8, 15].

Orientation of the circular orbits in the KdS spacetimes must be related to locally non-rotating frames (LNRF). Locally measured axial component of 4-momentum in the LNRF is given by

$$p^{(\varphi)} = mrLA^{-1/2}\,.$$

The circular orbits with $p^{(\varphi)} > 0$, $(L > 0)$ are co-rotating, and the circular orbits with $p^{(\varphi)} < 0$, $(L < 0)$ are counter-rotating, in agreement with the case of asymptotically flat Kerr spacetimes [16].

The circular geodesics can be astrophysically relevant, if they are stable with respect to radial perturbations. The loci of the stable circular orbits are given by $\mathrm{d}^2R/\mathrm{d}r^2 \geq 0$ that has to be satisfied simultaneously with the conditions $R(r) = 0$ and $\mathrm{d}R/\mathrm{d}r = 0$ determining the circular orbits. The radii of the stable orbits of both families are restricted by the condition [8]

$$r[6 - r + r^3(4r - 15)y] \mp 8a\left[r(1 - yr^3)^3\right]^{1/2} + a^2\left[3 + r^2y(1 - 4yr^3)\right] \geq 0\,.$$

A detailed analysis shows that the critical value of y for the existence of the stable plus-family orbits is given by $y_{\mathrm{crit(ms+)}} = 100/(5 + 2\sqrt{10}\,)^3 \doteq 0.06886$. No stable circular orbits (of any family) exist for $y > y_{\mathrm{crit(ms+)}}$. The critical value for the existence of the minus-family stable circular orbits is given by $y_{\mathrm{crit(ms-)}} = 12/15^4$. It coincides with the limit on the existence of the stable circular orbits in the SdS spacetimes [9].

For some special value of the specific angular momentum L_{mb}, the effective potential has two local maxima related by $E_{(+)}(r_{\mathrm{mb(i)}}; L_{\mathrm{mb}}, a, y) = E_{(+)}(r_{\mathrm{mb(o)}}; L_{\mathrm{mb}}, a, y)$ corresponding to the inner and outer marginally bound orbits. In the spacetimes with $y \geq 12/15^4$, the minus-family marginally bound orbits do not exist. In the spacetimes admitting stable plus-family orbits, there is $r_{\mathrm{mb(o)}} \sim r_{\mathrm{s}}$ but $r_{\mathrm{ms(o)}} \sim 0.7r_{\mathrm{s}}$.

The minus-family orbits have $L_- < 0$ in each KdS spacetime and such orbits are counter-rotating relative to the LNRF. In the black-hole spacetimes, the plus-family orbits are co-rotating in almost all radii where the circular orbits are allowed except some region in vicinity of the static radius, where they become to be counter-rotating. However, these orbits are unstable and the plus-family and minus-family orbits coalesce at the static radius.

Angular velocity $\Omega = \mathrm{d}\varphi/\mathrm{d}t$ of a thin, Keplerian accretion disc is given by

$$\Omega_{\mathrm{K}\pm} = \pm\frac{1}{\frac{r^{3/2}}{(1-yr^3)^{1/2}} \pm a}\,. \tag{5}$$

Matter in the thin disc spirals from the outer marginally stable orbit through the sequence of stable circular orbits down to the inner marginally stable orbit losing energy and angular momentum due to the viscosity. The necessary conditions for such a differential rotation $\mathrm{d}\Omega_{\mathrm{K}+}/\mathrm{d}r < 0$, $\mathrm{d}L_+/\mathrm{d}r \geq 0$ or $\mathrm{d}\Omega_{\mathrm{K}-}/\mathrm{d}r > 0$, $\mathrm{d}L_-/\mathrm{d}r \leq 0$ are fulfilled by the relations (4) and (5). The efficiency of accretion, i.e. the efficiency of conversion of rest mass into heat energy of any element of matter transversing the discs from their outer edge located on the outer marginally stable orbit to their inner edge located on the inner marginally stable orbit is given by $\eta \equiv E_{\mathrm{ms(o)}} - E_{\mathrm{ms(i)}}$. For Keplerian discs co-rotating around extreme KdS black holes, the accretion efficiency reaches maximum value of $\eta \sim 0.43$ for the pure Kerr case ($y = 0$) and tends to zero for $y \to y_{\mathrm{c(KdS)}} \doteq 0.059$, the maximum value of y admitting black holes.

Thick discs

Basic properties of thick discs are determined by equilibrium configurations of perfect fluid. Stress-energy tensor of perfect fluid is given by $T^{\mu}{}_{\nu} = (p + \varepsilon)U^{\mu}U_{\nu} + p\,\delta^{\mu}_{\nu}$, where ε and p denote total energy density and pressure of the fluid, U^{μ} is its four velocity. We shall consider test perfect fluid rotating in the φ direction, i.e., $U^{\mu} = (U^{t}, U^{\varphi}, 0, 0)$. The rotating fluid can be characterized by the vector fields of the angular velocity $\Omega(r, \vartheta)$ and the specific angular momentum $\ell(r, \vartheta)$, defined by $\Omega = U^{\varphi}/U^{t}$, $\ell = -U_{\varphi}/U_{t}$. Projecting the energy-momentum conservation law $T^{\mu\nu}{}_{;\nu} = 0$ onto the hypersurface orthogonal to the four velocity U^{μ} by the projection tensor $h_{\mu\nu} = g_{\mu\nu} + U_{\mu}U_{\nu}$, we obtain the relativistic Euler equation in the form

$$\frac{\partial_{\mu} p}{p + \varepsilon} = -\partial_{\mu}(\ln U_{t}) + \frac{\Omega\,\partial_{\mu}\ell}{1 - \Omega\ell}\,, \quad (U_{t})^{2} = \frac{g_{t\varphi}^{2} - g_{tt}\,g_{\varphi\varphi}}{g_{\varphi\varphi} + 2\ell g_{t\varphi} + \ell^{2} g_{tt}}\,.$$

For barytropic perfect fluid with an equation of state $p = p(\varepsilon)$, the solution of the relativistic Euler equation can be given by Boyer's condition determining the surfaces of constant pressure through the „equipotential surfaces" of the potential $W(r, \vartheta)$ by the relations [7]

$$\int_{0}^{p} \frac{\mathrm{d}p}{p + \varepsilon} = W_{\mathrm{in}} - W = \ln(U_{t})_{\mathrm{in}} - \ln(U_{t}) + \int_{\ell_{\mathrm{in}}}^{\ell} \frac{\Omega\,\mathrm{d}\ell}{1 - \Omega\ell}\,. \tag{6}$$

The equipotential surfaces are determined by the condition $W(r, \vartheta) = \mathrm{const}$, and in a given spacetime can be found from Eq. (6), if a rotation law $\Omega = \Omega(\ell)$ is given. Equilibrium configurations of test perfect fluid are determined by the equipotential surfaces which can be closed or open. Moreover, there is a special class of critical, self-crossing surfaces (with a cusp), which can be either closed or open. The closed equipotential surfaces determine stationary toroidal configurations. The fluid can fill any closed surface – at the surface of the equilibrium configuration pressure vanishes, but its gradient is non-zero [6]. On the other hand, the open equipotential surfaces are important in dynamical situations, e.g., in modeling of jets [17, 18]. The critical, self-crossing closed equipotential surfaces W_{cusp} are important in the theory of thick accretion discs, because accretion onto the black hole through the cusp of the equipotential surface is possible due to a little overcoming of the critical equipotential surface by the surface of the disc (Paczyński mechanism) [6]. All characteristic properties of the equipotential surfaces for a general rotation law are reflected by those of the marginally stable configurations with $\ell(r,\vartheta) = \mathrm{const}$, see [19, 20]. Then $W(r,\vartheta) = \ln U_t(r,\vartheta)$.

The equipotential surfaces $\vartheta = \vartheta(r)$ are given by the relation

$$\frac{\mathrm{d}\vartheta}{\mathrm{d}r} = -\frac{\partial p/\partial r}{\partial p/\partial \vartheta} = -\frac{\partial U_t/\partial r}{\partial U_t/\partial \vartheta}\,.$$

In the KdS spacetimes there is

$$W(r,\vartheta) = \ln \frac{\varrho \Delta_r^{1/2} \Delta_\vartheta^{1/2} \sin\vartheta}{I\left[\Delta_\vartheta \sin^2\vartheta (r^2+a^2-a\ell)^2 - \Delta_r(\ell - a\sin^2\vartheta)^2\right]^{1/2}}\,.$$

The best insight into the $\ell = \mathrm{const}$ configurations is given by properties of $W(r,\vartheta)$ in the equatorial plane ($\vartheta = \pi/2$). Condition for the local extrema of the potential $W(r,\vartheta = \pi/2)$ is identical with the condition of vanishing of the pressure gradient ($\partial U_t/\partial r = 0 = \partial U_t/\partial\vartheta$). Since in the equatorial plane there is $\partial U_t/\partial \vartheta = 0$, independently of $\ell = \mathrm{const}$, the only relevant condition is $\partial U_t/\partial r = 0$, which implies the relation $\ell = \ell_{\mathrm{K}\pm}(r;a,y)$ with $\ell_{\mathrm{K}\pm}$ being the specific angular momentum of the geodetical Keplerian orbits

$$\ell_{\mathrm{K}\pm}(r;a,y) \equiv \pm\frac{(r^2+a^2)(1-yr^3)^{1/2} \mp ar^{1/2}[2+yr(r^2+a^2)]}{r^{3/2}[1-y(r^2+a^2)] - 2r^{1/2} \pm a(1-yr^3)^{1/2}}\,.$$

The closed equipotential surfaces, and surfaces with a cusp allowing the outflow of matter from the disc, are permitted in those parts of the functions $\ell_{\mathrm{K}\pm}(r;a,y)$ enabling the existence of stable circular geodesics, $\ell \in (\ell_{\mathrm{ms(i)}}, \ell_{\mathrm{ms(o)}})$, where the centre of the equilibrium configurations is located. We can distinguish three kinds of discs:

Accretion discs: $\ell \in (\ell_{\mathrm{ms(i)}}, \ell_{\mathrm{mb}})$; the last closed surface is self-crossing in the inner cusp, another critical surface self-crossing in the outer cusp is open.

Marginally bound accretion discs: $\ell = \ell_{\mathrm{mb}}$; the last closed surface is self-crossing in both the inner and the outer cusp.

Excretion discs: $\ell \in (\ell_{\mathrm{mb}}, \ell_{\mathrm{ms(o)}})$; the last closed surface is self-crossing in the outer cusp, another critical surface self-crossing in the inner cusp is open.

Conclusions

For astrophysically relevant black holes ($M < 10^{12}\,\mathrm{M}_\odot$) and the observed RRCC, the cosmological parameter is so small ($y < 10^{-22}$) that both co-rotating and counter-rotating discs can exist around KdS black holes. The efficiency of the accretion process is then extremely close to the values relevant for Kerr black holes being strongest for thin discs orbiting extreme black holes. It is suppressed for a descending and/or for $\ell = \mathrm{const}$ growing from $\ell_{\mathrm{ms(i)}}$ up to ℓ_{mb}.

The crucial effects caused by the RRCC are illustrated in Fig. 1.

- The outer edge of the discs. Its presence nearby the static radius enables outflow of mass and angular momentum from the discs due to a violation of mechanical equilibrium stabilizing the runaway instability [21] – such an outflow is impossible from discs around isolated black holes in asymptotically flat spacetimes [6].

- Strong collimation effect on jets escaping along the rotational axis of toroidal discs indicated by open equipotential surfaces that are narrowing strongly after crossing the static radius.

We can give to our results the proper astrophysical relevance by presenting numerical estimates for observationally established current value of the relict repulsive cosmological constant $\Lambda_0 \approx 1.3 \times 10^{-56}\,\mathrm{cm}^{-2}$.[1] The basic characteristics are given for both the thin and thick discs (see Table 1). Outer edge of the marginally bound thick accretion disc is determined by the outer marginally bound circular orbit which is located almost at the static radius of a given spacetime. Dimensions of accretion discs around stellar-mass black holes ($M \sim 10\,\mathrm{M}_{\odot}$) in binary systems are typically 10^{-3} pc, dimensions of large galaxies with central black-hole mass $M \sim 10^{8}\,\mathrm{M}_{\odot}$, of both spiral and elliptical type, are in the interval 50–$100\,\mathrm{kpc}$, and the extremely large elliptical galaxies of cD type with central black-hole mass $M \sim 3 \times 10^{9}\,\mathrm{M}_{\odot}$ extend up to $1\,\mathrm{Mpc}$ [4]. Therefore, the influence of the RRCC is quite negligible in binary systems of stellar-mass black holes, but it can be relevant for accretion discs in large galaxies as the static radius puts limit on the extension of the discs well inside the galaxies. Moreover, the agreement (up to one order) of the dimension of the static radius related to the mass parameter of central black holes at nuclei of large galaxies with extension of such galaxies suggests that the RRCC could play an important role in formation and evolution of these galaxies. The first step in confirming such a suggestion is modeling of self-gravitating discs with a pseudo-Newtonian potential having in spherical spacetimes simple form $\psi = -(1 + \frac{1}{2}yr^3)/(r - 2 + yr^3)$ [23]. The Machian effects of the vacuum energy are thus reflected both at the universe and the galactic scales.

Acknowledgments

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[1] For more detailed information see Ref. [22], where the estimates for primordial black holes in the early universe with a repulsive cosmological constant related to a hypothetical vacuum energy density connected with the electroweak symmetry breaking or the quark confinement are presented.

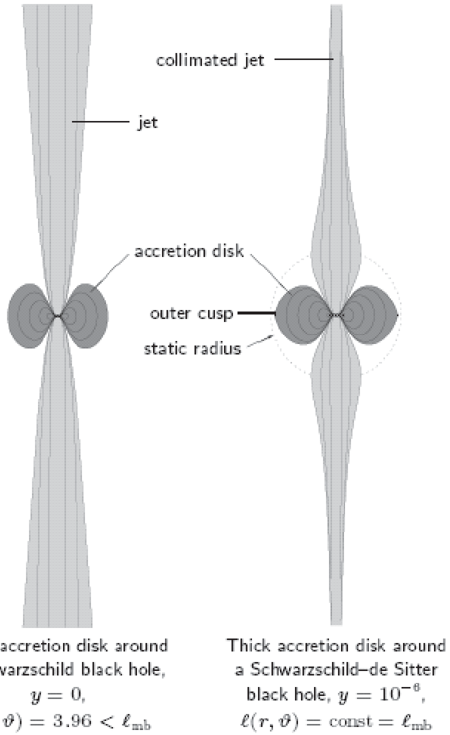

Figure 1: Shapes of thick discs and collimation of jets due to a cosmic repulsion. The effect of collimation is relevant near the static radius and further. Left picture depicts thick accretion discs orbiting the Kerr black hole ($y = 0$, $a^2 = 0.99$; $\ell \approx \ell_{\mathrm{mb}}$) and the Schwarzschild black hole ($y = 0$, $a = 0$; $\ell \approx \ell_{\mathrm{mb}}$), right picture depicts thick marginally bound accretion discs orbiting the KdS black hole ($y = 10^{-6}$, $a^2 = 0.99$; $\ell = \ell_{\mathrm{mb}}$) and the SdS black hole ($y = 10^{-6}$, $a = 0$; $\ell = \ell_{\mathrm{mb}}$).

y	10^{-44}	10^{-34}	10^{-30}	10^{-28}	10^{-26}	10^{-22}
$M/\mathrm{M}_{\odot}$	10	10^6	10^8	10^9	10^{10}	10^{12}
$r_{\mathrm{s}}/[\mathrm{kpc}]$	0.2	11	50	110	230	1100
$r_{\mathrm{ms}}/[\mathrm{kpc}]$	0.15	6.7	31	67	150	670

Table 1: Mass parameter, the static radius and radius of the outer marginally stable orbit in extreme KdS black-hole spacetimes are given for the RRCC indicated by recent cosmological observations.

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